**Math Challenge 1.** Prove that for every choice of real numbers $a, b, c, d > 0$,

$$\sqrt{ab} + \sqrt{bc} + \sqrt{cd} + \sqrt{da} \le a + b + c + d.$$

***Solution.*** Let *x* and *y* be two positive integers. Then $0 \le (x - y)^2 = x^2 - 2xy + y^2$. Adding 4*xy* to both sides of the inequality, we obtain $4xy \le x^2 + 2xy + y^2$, which is equivalent to $4xy \le (x + y)^2$. Thus,

$$2\sqrt{xy} \le x + y \quad \text{for all positive numbers } x \text{ and } y.$$

Because $a, b, c, d > 0$, we have in particular,

$$2\sqrt{ab} \le a + b,$$

$$2\sqrt{bc} \le b + c,$$

$$2\sqrt{cd} \le c + d,$$

$$2\sqrt{da} \le d + a.$$

Summing the inequalities yields

$$2\sqrt{ab} + 2\sqrt{bc} + 2\sqrt{cd} + 2\sqrt{da} \le 2a + 2b + 2c + 2d.$$

Finally, dividing both sides by 2, we obtain

$$\sqrt{ab} + \sqrt{bc} + \sqrt{cd} + \sqrt{da} \le a + b + c + d.$$