**Math Challenge 7.** Without the use of a computer or calculator, solve the following equation for x.

$$2\log_x e - \log_{ex} e - 3\log_{e^2x} e = 0$$

***Solution***. First, note that the base of a logarithm must be positive and cannot be 1. Thus, we assume $x > 0$, $x \neq 1$, $x \neq e^{-1}$, $x \neq e^{-2}$. Using the change-of-base formula, we may rewrite the given equation as

$$2\frac{\ln e}{\ln x} - \frac{\ln e}{\ln(ex)} - 3\frac{\ln e}{\ln(e^2x)} = 0.$$

Using the properties of logarithms and that $\ln e = 1$, we may further write

$$\frac{2}{\ln x} - \frac{1}{1 + \ln x} - \frac{3}{2 + \ln x} = 0.$$

Setting $u = \ln x$, we obtain $\frac{2}{u} - \frac{1}{1+u} - \frac{3}{2+u} = 0.$ Note that all the denominators are nonzero by the above assumption, so we multiply both sides of the equation by $u(1+u)(2+u)$ to obtain

$$2(1+u)(2+u) - u(2+u) - 3u(1+u) = 0 \quad \Leftrightarrow \quad 2u^2 - u - 4 = 0.$$

Using the quadratic formula, we have $u = (1 \pm \sqrt{33})/4$. Because $u = \ln x$, the solutions are $x = e^{(1+\sqrt{33})/4}$ and $x = e^{(1-\sqrt{33})/4}$.