Math Challenge 8. Without the use of a computer or calculator, find the sum of the digits of N, where

$$N = 9 + 99 + 999 + \ldots + \underbrace{999\ldots9}_{999 \text{ digits}}.$$

Solution.

$$\begin{aligned}
N &= (10-1) + (100-1) + (1000-1) + \ldots + (\underbrace{1000\ldots0}_{1000 \text{ digits}} - 1) \\
&= (10^1 - 1) + (10^2 - 1) + (10^3 - 1) + \ldots + (10^{999} - 1) \\
&= \sum_{n=1}^{999} (10^n - 1) \\
&= \sum_{n=1}^{999} 10^n - \sum_{n=1}^{999} 1 \\
&= \left[\sum_{n=0}^{999} 10^n\right] - 1 - 999 \\
&= \frac{10^{1000} - 1}{10 - 1} - 1000 \\
&= \frac{\overbrace{1000\ldots00000}^{1001 \text{ digits}} - 1}{9} - 1000 \\
&= \frac{\overbrace{999\ldots99999}^{1000 \text{ digits}}}{9} - 1000 \\
&= \overbrace{111\ldots11111}^{1000 \text{ digits}} - 1000 \\
&= \overbrace{111\ldots1}^{996 \text{ digits}}0111.
\end{aligned}$$

There are 999 ones and 1 zero in N. Thus, the sum of the digits of N is equal to 999.