

Math Challenge 11. Peggy’s calculator has been malfunctioning. The screen does not display the numbers that she enters, only the results from the calculation. Further, the calculator has a random error that changes a 7 to an 8 with probability $1/6$ and changes an 8 to a 7 with probability $1/4$. Peggy is going to enter

$$878 - 878$$

in this calculator. If all other keys are functioning correctly, what is the probability that she finds a nonzero answer?

Solution. All possible enterings for a “zero” answer and their probabilities:

$$P\left(\boxed{7}\boxed{7}\boxed{7} - \boxed{7}\boxed{7}\boxed{7}\right) = \left(\frac{1}{4} \cdot \frac{5}{6} \cdot \frac{1}{4}\right)^2 = \frac{25}{9216}$$

$$P\left(\boxed{7}\boxed{7}\boxed{8} - \boxed{7}\boxed{7}\boxed{8}\right) = \left(\frac{1}{4} \cdot \frac{5}{6} \cdot \frac{3}{4}\right)^2 = \frac{225}{9216}$$

$$P\left(\boxed{7}\boxed{8}\boxed{7} - \boxed{7}\boxed{8}\boxed{7}\right) = \left(\frac{1}{4} \cdot \frac{1}{6} \cdot \frac{1}{4}\right)^2 = \frac{1}{9216}$$

$$P\left(\boxed{7}\boxed{8}\boxed{8} - \boxed{7}\boxed{8}\boxed{8}\right) = \left(\frac{1}{4} \cdot \frac{1}{6} \cdot \frac{3}{4}\right)^2 = \frac{9}{9216}$$

$$P\left(\boxed{8}\boxed{7}\boxed{7} - \boxed{8}\boxed{7}\boxed{7}\right) = \left(\frac{3}{4} \cdot \frac{5}{6} \cdot \frac{1}{4}\right)^2 = \frac{225}{9216}$$

$$P\left(\boxed{8}\boxed{7}\boxed{8} - \boxed{8}\boxed{7}\boxed{8}\right) = \left(\frac{3}{4} \cdot \frac{5}{6} \cdot \frac{3}{4}\right)^2 = \frac{2025}{9216}$$

$$P\left(\boxed{8}\boxed{8}\boxed{7} - \boxed{8}\boxed{8}\boxed{7}\right) = \left(\frac{3}{4} \cdot \frac{1}{6} \cdot \frac{1}{4}\right)^2 = \frac{9}{9216}$$

$$P\left(\boxed{8}\boxed{8}\boxed{8} - \boxed{8}\boxed{8}\boxed{8}\right) = \left(\frac{3}{4} \cdot \frac{1}{6} \cdot \frac{3}{4}\right)^2 = \frac{81}{9216}$$

Then

$$P(\text{a zero answer}) = \frac{25}{9216} + \frac{225}{9216} + \frac{1}{9216} + \frac{9}{9216} + \frac{225}{9216} + \frac{2025}{9216} + \frac{9}{9216} + \frac{81}{9216} = \frac{2600}{9216}.$$

$$\text{Thus, } P(\text{a nonzero answer}) = 1 - \frac{2600}{9216} = \frac{827}{1152} \approx 71.8\%.$$